A Sparse Decomposition of Low Rank Symmetric Positive Semi-Definite Matrices

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Suppose that $A \in \mathbb{R}^{N\times N}$ is symmetric positive semidefinite with rank $K \leq N$. Our goal is to decompose A into K rank-one matrices $\sum_{k=1}^{K} g_k g_k^T$ and the modes $\{g_k\}_{k=1}^{K}$ are required to be as sparse as possible. In contrast to eigen decomposition, these sparse modes are not required to be orthogonal. Such a problem arises in random field parametrization where A is the covariance function and is intractable to solve in general. In this paper, we partition the indices from 1 to N into several patches and propose to quantify the sparseness of a vector by the number of patches on which it is nonzero, which is called patch-wise sparseness. Our aim is to find the decomposition which minimizes the total patch-wise sparseness of the decomposed modes. We propose a domain-decomposition type method, called intrinsic sparse mode decomposition (ISMD), which follows the "local-modes-construction + patching-up" procedure. The key step in ISMD is to construct local pieces of the intrinsic sparse modes by a joint diagonalization problem. Thereafter a pivoted Cholesky decomposition is utilized to glue these local pieces together. Optimal sparse decomposition, consistency with different domain decomposition and robustness to small perturbation are proved under the so called regular sparse assumption. We provide simulation results to show the efficiency and robustness of the ISMD. We also compare ISMD to other existing methods, e.g., eigen decomposition, pivoted Cholesky decomposition and convex relaxation of sparse principle component analysis.